



Research article

DEVELOPMENT OF AN ENHANCED LAGRANGE POLYNOMIAL MODEL FOR RELIABILITY ASSESSMENT OF POWER DISTRIBUTION SYSTEMS

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Abstract

Distribution system is one of the three major reliability levels of power system. Other levels include generation and transmission systems. Generation system produces the electrical energy needed for use while transmission system forms a link between generation system and distribution system. Distribution system delivers the transmitted energy to the end users i.e. the customers. This paper develops an enhanced Lagrange polynomial model for reliability assessment of power distribution system using five selected distribution systems as case studies. The system reliability indices were identified and computed using appropriate mathematical notations of failure rate analysis. The mean values of the system average interruption duration index (SAIDI) and system average interruption frequency index (SAIFI) were used for the development of the relative customer average interruption duration index (CAIDI) model with the aid of curve fitting analysis. Ten years outage data from five major cities in Nigeria- Ibadan, Ilorin, Ikeja, Port-Harcourt and Kaduna were collected and analyzed for use. This model is subject to Lagrange polynomial function due to the polynomial nature of the model. Appropriate mathematical notations of polynomial functions were employed to develop the enhanced Lagrange polynomial function model which is an improvement over the Lagrange polynomial model.

The result of the enhanced Lagrange polynomial function model developed reveals details about the reliability levels of the distribution systems. The reliability level of Kaduna distribution system appreciably improved with an enhanced polynomial coefficient of 1.05312 as a result of the level of industrial activities in the city compared to other cities used as case studies in the paper which ranks the distribution system as the most reliable. Ibadan, Ilorin, Ikeja and Port-Harcourt distribution systems have enhanced polynomial coefficients of 0.95393, -0.88313, 0.82293 and 0.06342 respectively, thus ranking Ilorin distribution system as the least reliable. This is due to the intermittent interruptions experienced by most of the customers attached to this distribution



system. The knowledge of these coefficients will form a good basis for adequate planning and management of power distribution systems. **Copyright © IJRETR, all rights reserved. USA**

Keywords: Enhanced Lagrange Polynomial, Model, Outage, Reliability, SAIDI, SAIFI, CAIDI, Relative CAIDI.

I. Introduction

Power system reliability assessment is an integral element in determining the facilities required in a power system to satisfy the load requirements in a satisfactory and continuous manner. Reliability studies and the development of reliability models and tools are very important activities in the design and operation of reliable power systems. Reliability is considered to be a key element in power system operation and planning. Power distribution systems are directly linked to consumers. Thus, the distribution system plays an important role in the overall power system reliability and the perceived reliability to customers. By improving distribution protection systems such as sustained outage times, power system reliability can be enhanced (Allan and Da-Silva 1995, Rausand and Hoyland 2004, Gaver *et al* 2007).

Electric power system has a primary role of providing reliable and continuous supply of electric energy to satisfy system load (Billinton and Aboreshaid 1998, Chery *et al* 2003).

Electric power system reliability, in a broad sense, can be defined as the ability of the system to provide an adequate supply of electric power with a satisfactory quality. Accurate reliability analysis of power systems helps to predict future failure behavior and make appropriate maintenance plans (Endrenyi and Anders 2006; Endrenyi *et al* 2008). Reliability performance of distribution utilities has received considerable attention in recent years (Elena *et al* 2010, Danny 1989, Enderenyi *et al* 2008).

The reliability of power distribution systems is greatly affected by outages caused by different environmental factors on overhead lines. Since animals cause significant number of outages on overhead distribution systems, it is important to investigate these outages (Rios *et al* 1998, Gangel and Ringlee 2005, Farag *et al* 1998).

Distribution networks are normally meshed in design but the operation is nearly always configured radially. Re-configuration consists of changing the network configuration by opening/closing feeders and tie switches so that the networks become radial in operation. The configuration of distribution networks may be modified manually or by automatic switching operations for supplying the loads aiming at minimizing the cost of active power losses. Likewise, reconfiguration may increase system security and power quality (Billinton and Wang 1998, Elena *et al* 2010, Dan 2003, Enrico and Gianfranco 2004, Nagaraj *et al* 2004).

Review of Related Works

Nagaraj *et al* (2004) presented modeling and analysis of distribution reliability indices. In this paper, an efficient Monte Carlo simulation method for distribution system reliability assessment was presented. Analysis of outage data from a practical distribution system was performed to determine the failure and repair models appropriate for use in the Monte Carlo simulation. The sensitivity of the reliability indices to the choice of model is presented. Finally, the impact of protection devices on the statistical distribution of SAIFI for a practical distribution feeder is presented.

Mahmud and Saeed (2009) presented reliability analysis in electrical distribution system considering preventive maintenance applications on circuit breakers. The paper presented the results of the preventive maintenance application based study and modeling of failure rates in breakers of electrical distribution system which is a critical issue in the reliability assessment of a system. In the analysis considered in this paper, the impacts of failure rate variations caused by a preventive maintenance are examined. This is considered as a part of a Reliability Centered Maintenance (RCM) application program. A number of load point reliability indices are derived using the mathematical model of the failure rate which is established using the observed data in a distribution system.

Dan (2003) presented a reliability analysis algorithm for large scale radially operated (with respect to substation), reconfigurable, electrical distribution systems. The algorithm takes into account equipment power



handling constraints and converges in a matter of seconds on systems containing thousands of components. Linked lists of segments are employed in obtaining the rapid convergence. A power flow calculation is used to check the power handling constraints. The placement of distributed generation and its effect on reliability is investigated. The work presented here evaluates improvement in reliability over a time-varying load curve. Reliability indices for load points and the overall system have been developed.

Enrico and Gianfranco (2004) presented a new approach for computing the probability distributions of reliability indices of large distribution systems under the usual hypotheses concerning the fault occurrence and considering the restoration times as random variables with any type of probability distribution. The proposed approach is based on the characteristic functions, which are directly connected to central moments and exploits the properties of the Poisson process to avoid performing the convolutions of the Probability Distribution Functions (PDFs).

Assessment of distribution system reliability using time series is done by Adejumobi (2005). The three-box-Jenkins forecasting models for time series (i.e Auto-regressive, moving-average and mixed auto-regressive moving average models) were considered in fault forecasting. The choice of a model was made from the computed samples autocorrelation and partial auto-correlation of the data. The advantage of a chosen model was determined by comparing the computed chi-square with standard statistic table. Forecast results were used to compute system reliability. Evaluation results show that hidden failures in protection system can downgrade the system reliability level as a result of the outages of equipment following the initial system disturbances.

A linear contribution factor model of distribution reliability indices and its application in Monte Carlo simulation and sensitivity analysis was reported by Fangxing *etal* (2003). A mathematical model for improvement in reliability indices was developed. This linear model can be applied to risk analysis and sensitivity analysis. Traditional approaches for both analyses require many repetitions of reliability index assessment. The model failed to appreciably improve the reliability indices of most system indices in an electrical distribution system of the National grid.

Meysam and Hasan (2009) proposed a market model where the reserve capacity is scheduled based on the desired risk level of customer; that is recognized by the expected load not supplied (ELNS) index. The reliability constraint in the objective function of reserve procurement is formulated as a function of generation and reserve allocation running the generating units as well as their failure probability. The reserve requirement is determined regarding transmission constraints and load containments due to transmission congestion.

II. Materials and Methods

The following steps were taken in analyzing the method used

- i. The system reliability indices for the distribution system under study was identified
- ii. The failure rate λ for the selected feeders on the distribution systems was computed using the relation

$$\lambda = \frac{N}{t - t'} \quad 1$$

Where:

N = the number of faults,

t and t' represent total time and down time respectively.

- iii. The system reliability indices – SAIDI, SAIFI and CAIDI were computed.
- (a) System Average Interruption Duration Index, SAIDI is given by

$$\begin{aligned} SAIDI &= \frac{\text{Customer interruption durations}}{\text{Total Number of Customers Served}} \\ &= \frac{\sum_{i=1}^n r_i N_i}{\sum_{i=1}^n N_T} \quad 2 \end{aligned}$$

- (b) System Average Interruption Frequency Index, SAIFI is given by



$$SAIFI = \frac{\text{Total number of customer int eruptions}}{\text{Total Number of Customers Served}}$$

$$= \frac{\sum_{i=1}^n N_i}{\sum_{i=1}^n N_T} \quad 3$$

(c) Customer Average Interruption Duration Index, CAIDI is given by

$$CAIDI = \frac{\text{Customer int eruption durations}}{\text{Total Number of Customers Interruption}}$$

$$= \frac{\sum_{i=1}^n r_i N_i}{\sum_{i=1}^n N_i} \quad 4$$

where:

- r_i = Restoration time for each interruption for the i^{th} customer.
- N_i = Number of interrupted customers for each interruption event during reporting period.
- N_T = Total number of customers served for area being indexed.

iv. Compute \overline{SAIDI} , \overline{SAIFI}

v. Compute $RC = \frac{\overline{SAIDI}}{\overline{SAIFI}}$ 5

vi. Plot \overline{SAIDI} Vs \overline{SAIFI} to get polynomial equation.

vii. Note the various coefficients.

viii. Subject the model to Lagrange polynomial functions.

ix. Obtain Lagrange coefficients

x. Compare the coefficients

Development of an Enhanced Lagrange Polynomial Model

Consider a given set of $N+1$ data points, $\{(x_0, y_0), (x_1, y_1), \dots, (x_N, y_N)\}$

The coefficients of an N th-degree polynomial function to match the data points can be found.

$$P_N(x) = a_0 + a_1x + a_2x^2 + \dots + a_Nx^N \quad 6$$

To obtain the coefficients, the following system of equations were used.

$$a_0 + x_0a_1 + x_0^2a_2 + \dots + x_0^Na_N = y_0 \quad 7$$

$$a_0 + x_1a_1 + x_1^2a_2 + \dots + x_1^Na_N = y_1 \quad 8$$

⋮

$$a_0 + x_Na_1 + x_N^2a_2 + \dots + x_N^Na_N = y_N \quad 9$$

The values of the unknown functions at the points inside/outside the range of collected data points were estimated to get the interpolation/extrapolation.



As the number of data points increases, the number of unknown variables and equations increases, thus making it difficult to solve. In this case, alternatives were used to get the coefficients (a_0, a_1, \dots, a_n). Here, one of the methods is to use the Lagrange polynomials which can be written as

$$L_N(x) = y_0 \frac{(x-x_1)(x-x_2)\dots(x-x_N)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_N)} + y_1 \frac{(x-x_0)(x-x_2)\dots(x-x_N)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_N)} + \dots + y_N \frac{(x-x_0)(x-x_1)\dots(x-x_{N-1})}{(x_N-x_0)(x_N-x_1)\dots(x_N-x_{N-1})}$$

$$L_N(x) = \sum_{m=0}^N y_m L_{N,m}(x)$$

with

$$L_{N,m}(x) = \frac{\prod_{k \neq m}^N (x - x_k)}{\prod_{k \neq m}^N (x_m - x_k)} = \prod_{k \neq m}^N \frac{x - x_k}{x_m - x_k} \quad 11$$

The N-degree polynomial function matching the given N+1 points is unique. Therefore, equation (9), having the coefficients obtained from equation (8) must be the same as the Lagrange polynomial in equation (10).

The MATLAB routine "Lagranp()" which finds the coefficients of Lagrange polynomial together with each Lagrange coefficient polynomial $L_{N,m}(x)$ is used. This will be achieved by developing an algorithm for MATLAB programme. Equation (11) is the enhanced Lagrange polynomial model.

III. Discussion of Results

The relationship between the coefficients of relative CAIDI model, Lagrange polynomial and the enhanced Lagrange polynomial model for Ibadan distribution system is illustrated in Figure 1. The relative CAIDI model is a polynomial function with coefficients of 0.0025218, -0.0084318, 0.10923, -0.68797, 2.1627, -3.1428 and 2.4357. When this model was subjected to Lagrange polynomial model, the coefficients are 0.00015932, -0.0098716, 0.21036, -0.71026, 2.2319, -3.207 and 2.5623. The enhanced Lagrange polynomial function model has actually improved the coefficients to 0.00025221, -0.0084321, 0.10925, -0.68801, 2.1623, -3.1429 and 2.4359 making these coefficients to compare favorably well with the relative CAIDI model for Ibadan distribution system. This is probably due to the level of industrialization in the city of Ibadan. This is comparable to any other industrialized cities in the world.

Figure 2 shows the variations of coefficients of relative CAIDI model, Lagrange polynomial model and the enhanced Lagrange polynomial model for Ilorin distribution system. The first coefficients of the relative CAIDI polynomial model in this distribution system is -6.95×10^{-4} which corresponds to coefficients of -6.35×10^{-4} and -6.94×10^{-4} in the Lagrange and enhanced Lagrange polynomial models respectively because of the electricity requirements of this distribution system which is probably in the small scale level. Other coefficient are 0.00213, -0.005651, 0.15483, -0.048646, 0.70479 and -0.17655 for the relative CAIDI polynomial model while the other coefficient are 0.00324, -0.03314, 0.21317, -0.53216, 0.70479 and -0.21624 for Lagrange model and -0.00215, -0.028213, 0.15216, -0.5814, 0.79418 and 0.16918 for the enhanced Lagrange polynomial model. With this enhanced Lagrange polynomial function model, the reliability level of this distribution system is ensured.

In Figure 3, the reliability assessment of Ikeja distribution system using the enhanced Lagrange polynomial model is discussed. The reliability of most of the feeders on this distribution system is improved judging from the enhanced polynomial model coefficients of 6.5063×10^{-3} , 0.0019136, 0.056314 and 0.50126 which marked a high level of improvement contrary to the polynomial coefficients of 6.7213×10^{-6} , 0.0016243, 0.062146 and 0.51628 in the Lagrange polynomial model and fundamental coefficients of 6.5063×10^{-3} , 0.0019179, 0.05426 and 0.49562 in the standard relative CAIDI polynomial model. This is because this distribution system probably falls into the medium scale level which has a range in terms of the power requirements which could be true for other classifications.

Figure 4 illustrates the relationship between the polynomial coefficients using the relative CAIDI model, Lagrange model and enhanced Lagrange polynomial model for Port-Harcourt distribution systems. Majority of the feeders on this distribution systems are relatively reliable with the use of an enhanced Lagrange polynomial function



model which is better illustrated with coefficients improvement of 0.00093126, 0.012316, 0.090246 and 0.7316 with enhanced Lagrange polynomial model as compared to polynomial coefficients of 0.00096216, 0.083196, 0.063146 and 0.9215 with Lagrange polynomial model as well as polynomial coefficient of 0.00094044, 0.053344, 0.091043 and 0.7507 with the standard relative CAIDI model. However, most of the feeders on this distribution system experienced little outage that lasted for a very short time period before the faults were cleared. This accounts for the high level of reliability of these selected feeders of the distribution systems. In this case, the orders of the enhanced Lagrange polynomial model increases due to the level of industrialization of Port-Harcourt distribution system. The more industrialized a city is, the higher the order of the polynomial. The order of the polynomial goes up for cities where industries in them are fully operational.

The assessment of reliability levels of Kaduna distribution system is shown in Figure 5. Electricity requirements of this city fall into the very large scale level. This is judged from the enhanced Lagrange polynomial coefficients of 6.702×10^{-5} , 0.0022146, -0.028213, 0.15216, -0.5814, 0.79418 and 0.16918 when compared with Lagrange polynomial coefficients of -0.832×10^{-5} , 0.0042653, -0.043214, 0.32461, -0.5926 and 0.26231 as well as polynomial coefficients of -0.77×10^{-5} , 0.00021901, -0.021901, 0.16929, -0.5993, 0.79365 and 0.15056 in the standard relative CAIDI model. This distribution system is highly reliable since customers attached to this distribution feeder experienced little interruptions which were cleared promptly due to the proper maintenance actions on this distribution system.

The reliability representation of the five selected distribution systems used as case studies in this research paper is shown in Figure 6. Kaduna distribution system emerged as the most reliable with an enhanced Lagrange polynomial coefficient of 1.05312, a Lagrange polynomial coefficient of 1.07123 and a relative CAIDI coefficient of 1.05391. This is due to the high level of industrial activities in this city compared to any other industrialized cities in the world. Ilorin distribution system is the least reliable with a poor level of reliability which is obvious from the enhanced Lagrange polynomial coefficient of -0.88313, a Lagrange polynomial coefficient of -0.98396 and a relative CAIDI coefficient of -0.87512. Most of the customers attached to this distribution system experienced intermittent outages that lasted for a long period of time before the faults were cleared, thus, leading to the poor level of industrial activities in the city.

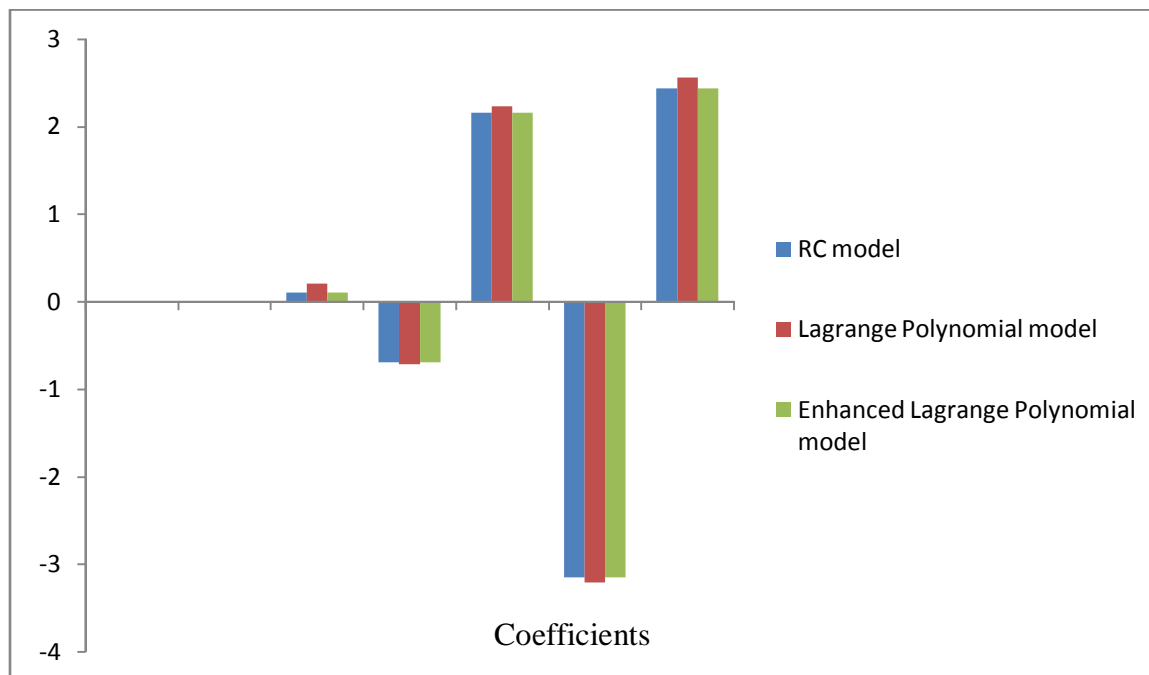


Figure 1: Coefficients of Ibadan Distribution System with the three models

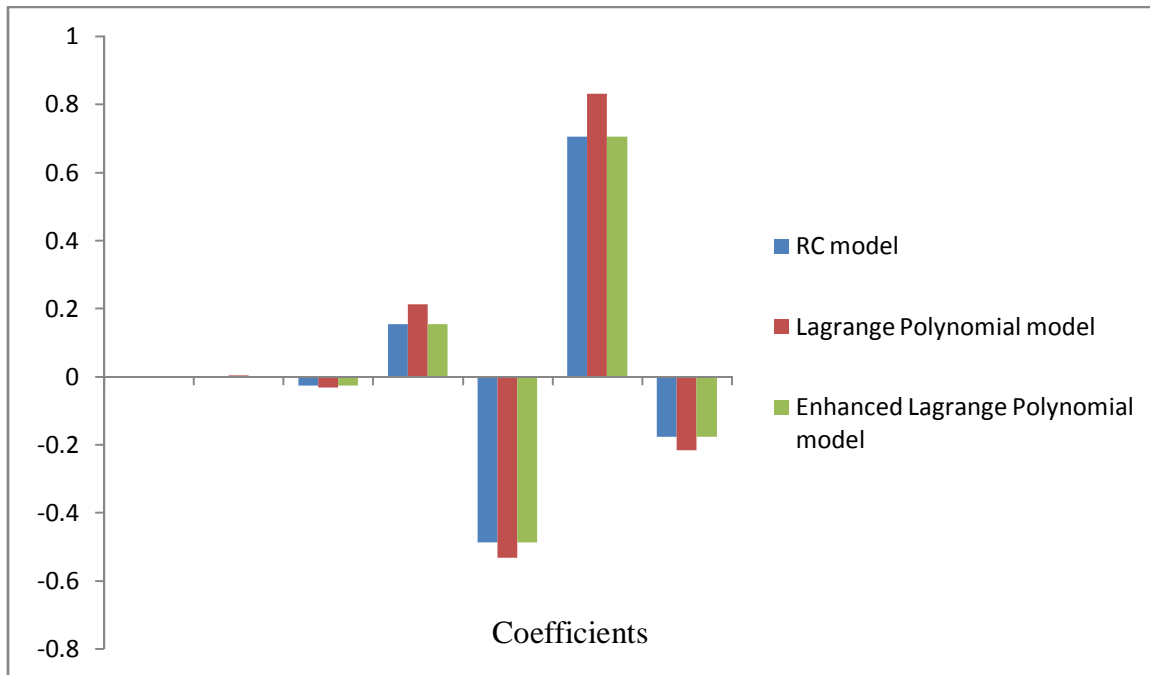


Figure 2: Coefficients of Ilorin Distribution System with the three models

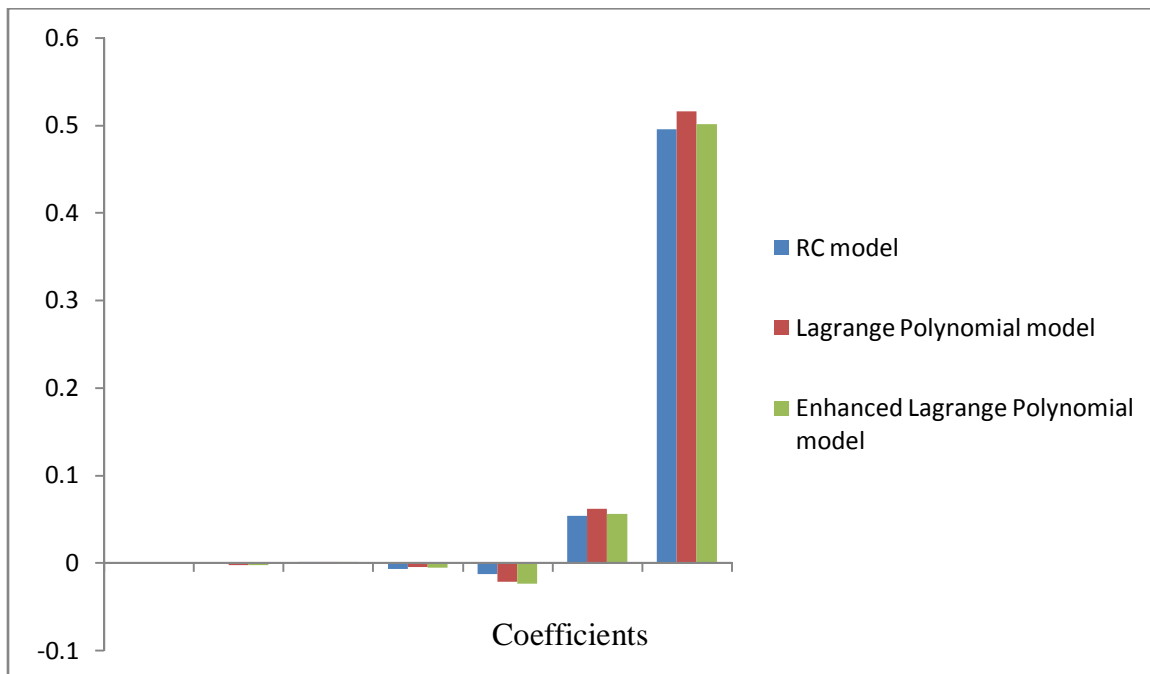


Figure 3: Coefficients of Ikeja Distribution System with the three models

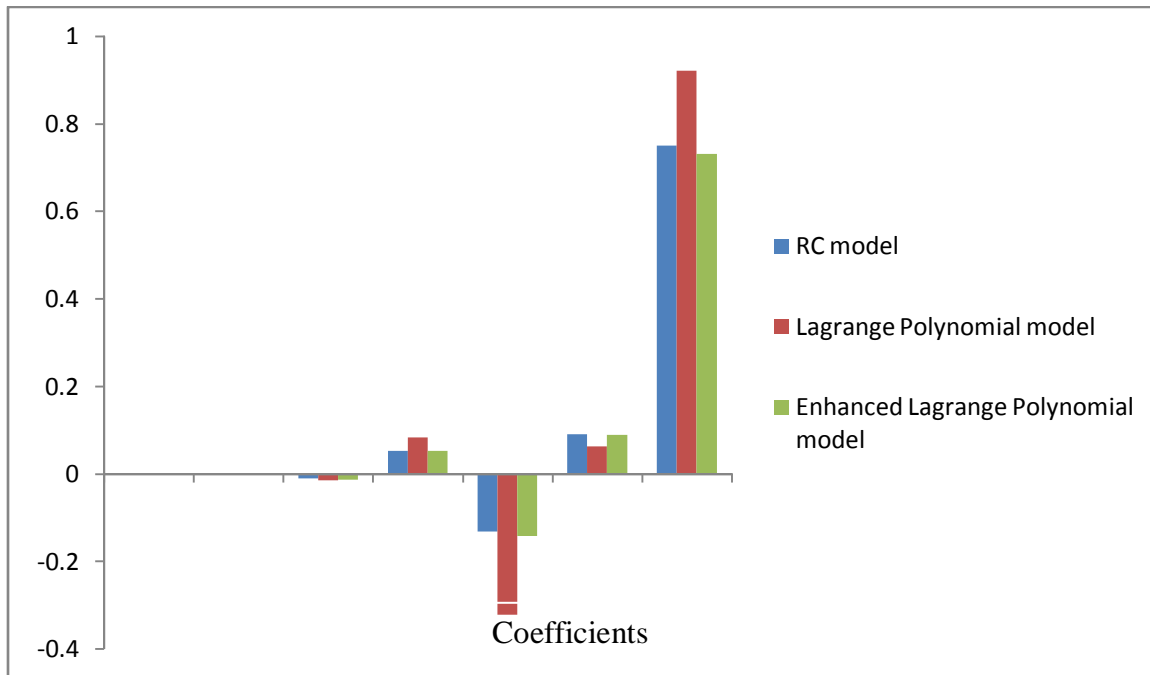


Figure 4: Coefficients of Port-Harcourt Distribution System with the three models

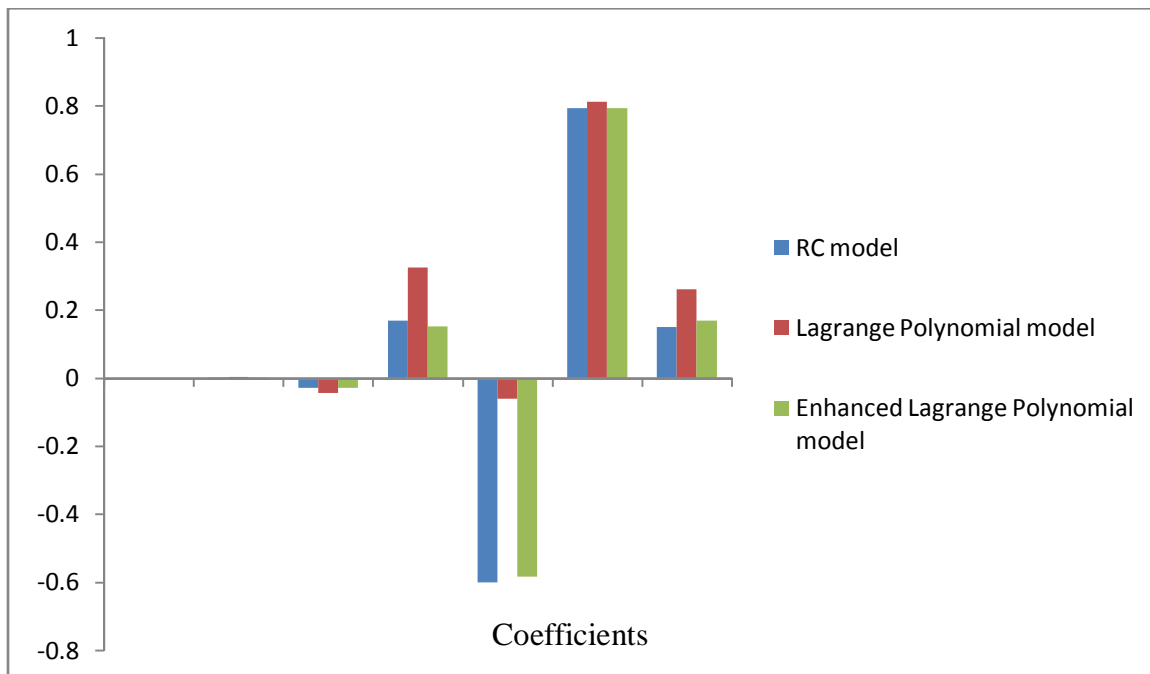


Figure 5: Coefficients of Kaduna Distribution System with the three models

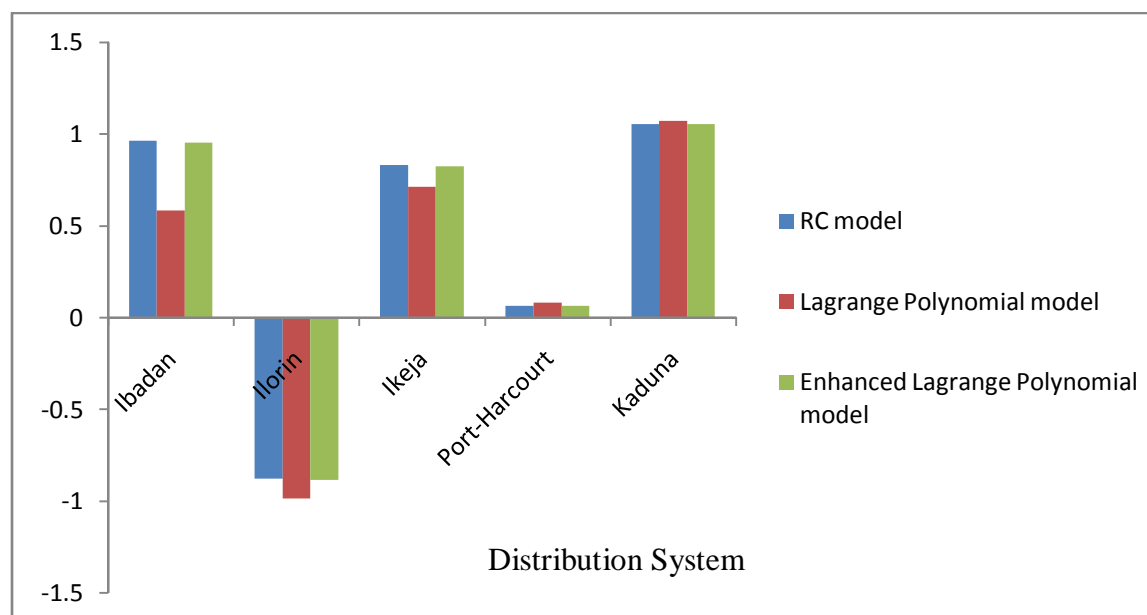


Figure 6: Coefficients of all Distribution System with the three models

IV. Conclusion

An enhanced Lagrange polynomial model for reliability assessment of power distribution systems have been presented. Five large typical cities in Nigeria- Ibadan, Ilorin, Ikeja, Port-Harcourt and Kaduna were selected as case studies in this research paper. The relative CAIDI model was first developed using ten years outage data from the distribution systems. This model was subjected to Lagrange polynomial function to develop the Lagrange polynomial function model. An enhanced Lagrange polynomial function model was then developed using appropriate mathematical notations. With the enhanced Lagrange polynomial function model, the reliability of Kaduna distribution system greatly improved with an enhanced polynomial coefficient of 1.05312 due to the level of industrial activities in the city compared to other industrialized cities in the world. Hence, this distribution emerged as the most reliable of all the distribution systems used as case studies in this research paper. Ilorin distribution system is the least reliable with an enhanced polynomial coefficient of -0.088313 as most of the customers attached to the distribution feeders experienced intermittent periods of power interruptions that put the customers in a period of prolonged darkness.

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